

# STFER FOR ANSYS WORKBENCH

Theory Manual

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## 1. OBJECT

STFER for Ansys Workbench is a software for post-processing finite element results to compute reinforcement ratios of concrete structures (such as slabs, walls, ...) that are modelled with shell or plate elements:

1. define combinations of load cases,
2. combine internal forces using these combinations,
3. compute reinforcement ratios.

This manual describes the methods used by STFER for Ansys Workbench version 1.5.0, which is used in the extension “Reinforcement Design V1.0”.

For guidance on how to use the software, refer to the User Manual [1].

For more information on this manual ask [support@stabilis.fr](mailto:support@stabilis.fr).

## 2. HISTORY OF CHANGES

- Version 1.5.0: First public version.

### **3. PROCESSING OF COMBINED RESULTS**

This chapter describes specific processing that may be done on finite element results before they are passed on the reinforcement computation algorithms.

#### **3.1. VOCABULARY**

*Load cases* can be either one step of the finite element calculation, or a list of steps.

*Groups* are logical groups of load cases or of other groups, with coefficients. The load cases can be summed ('+' operation), or can be mutually exclusive ('OR' operation), etc.

*Combinations* are defined in terms of load cases or groups and appropriate coefficients.

All of these are defined by the user (for details refer to the User Manual [1]).

From the combinations, STFER defines *elementary combinations* by considering all possibilities. Thus, depending on how the user defined the combinations, a single combination is often expanded into hundreds or thousands of elementary combinations.

#### **3.2. SYSTEMATIC CALCULATION**

This is a default method.

For each shell or plate element, the reinforcement computation algorithm is applied successively on all *elementary combinations*, and then the maximum reinforcement areas are computed.

#### **3.3. MINMAX ENVELOPE**

This method will be added in a future version to provide performance enhancements.

## 4. LONGITUDINAL REINFORCEMENTS

### 4.1. PLATE AND SHELL ELEMENTS

#### 4.1.1. Introduction

This chapter describes methods for the computation of steel reinforcements in plate and shell elements.

#### 4.1.2. Definitions and conventions

We consider a concrete shell or plate element with thickness  $t$ , whose local coordinate system is defined by in-plane orthogonal axes  $x$  and  $y$ , and normal axis  $z$ .

The generalized element forces and moments are expressed with regards to these axes, as shown in Figure 1:

- $N_{xx}$  and  $N_{yy}$ : normal forces per unit length, positive in tension;
- $N_{xy}$ : in-plane shear force per unit length;
- $M_{xx}$  and  $M_{yy}$ : bending moments per unit length. A positive value corresponds to a compression at the bottom face;
- $M_{xy}$ : twisting moment per unit length.  $M_{xy} > 0$  generates positive shear forces near the top face and negative shear forces near the bottom face;
- $Q_x$  and  $Q_y$ : generalized transverse shear forces per unit length.

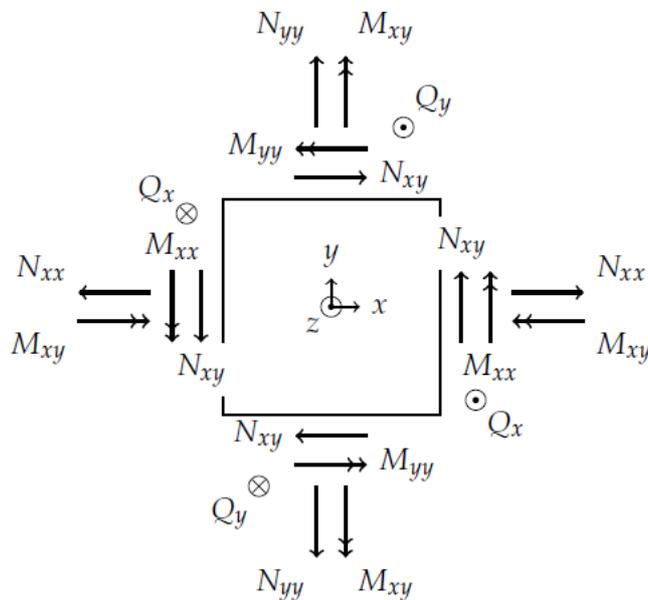


Figure 1 : Conventions for generalized forces and moments per unit length for shell and plate elements.

Generalized forces and moments are related to stress components by integration over the thickness of the shell:

$$N_{xx} = \int_{-t/2}^{t/2} \sigma_{xx} \cdot dz, \quad M_{xx} = \int_{-t/2}^{t/2} z \cdot \sigma_{xx} \cdot dz, \quad (1)$$

$$N_{yy} = \int_{-t/2}^{t/2} \sigma_{yy} \cdot dz, \quad M_{yy} = \int_{-t/2}^{t/2} z \cdot \sigma_{yy} \cdot dz, \quad (2)$$

$$N_{xy} = \int_{-t/2}^{t/2} \sigma_{xy} \cdot dz, \quad M_{xy} = \int_{-t/2}^{t/2} z \cdot \sigma_{xy} \cdot dz, \quad (3)$$

$$Q_x = \int_{-t/2}^{t/2} \sigma_{xz} \cdot dz, \quad Q_y = \int_{-t/2}^{t/2} \sigma_{yz} \cdot dz. \quad (4)$$

The subscript  $t$  (resp.  $b$ ) is used for variables related to the top (resp. bottom) face.

#### 4.1.3. Elements with linear stress profile

When the finite element model uses linear elements, the stress depends linearly on the position along the thickness of the shell (Figure 2). For example, the  $x$ -axis stress is given by:

$$\sigma_{xx}(z) = \frac{\sigma_{xx}^{top} + \sigma_{xx}^{bot}}{2} + \frac{z}{t} (\sigma_{xx}^{top} - \sigma_{xx}^{bot}). \quad (5)$$

The generalized forces and moments become:

$$N_{xx} = \frac{t}{2} (\sigma_{xx}^{top} + \sigma_{xx}^{bot}), \quad M_{xx} = \frac{t^2}{12} (\sigma_{xx}^{top} - \sigma_{xx}^{bot}), \quad (6)$$

$$N_{yy} = \frac{t}{2} (\sigma_{yy}^{top} + \sigma_{yy}^{bot}), \quad M_{yy} = \frac{t^2}{12} (\sigma_{yy}^{top} - \sigma_{yy}^{bot}), \quad (7)$$

$$N_{xy} = \frac{t}{2} (\sigma_{xy}^{top} + \sigma_{xy}^{bot}), \quad M_{xy} = \frac{t^2}{12} (\sigma_{xy}^{top} - \sigma_{xy}^{bot}), \quad (8)$$

$$Q_x = \frac{t}{2} (\sigma_{xz}^{top} + \sigma_{xz}^{bot}), \quad Q_y = \frac{t}{2} (\sigma_{yz}^{top} + \sigma_{yz}^{bot}). \quad (9)$$

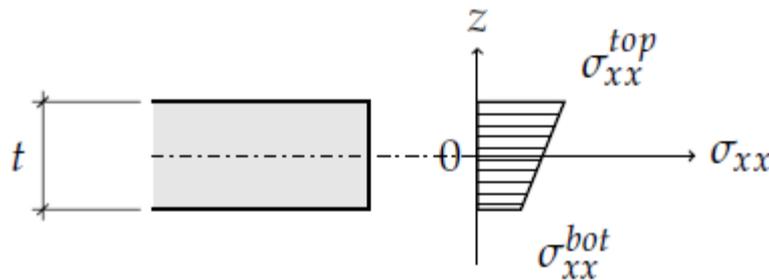


Figure 2 :  $x$ -axis membrane stress  $\sigma_{xx}$  with linear profile along the thickness of the shell element.

#### 4.1.4. Extracting finite element results

STFER uses generalized element forces and moments, extracted from the results of the finite element calculation. They are computed as the mean of the results at Gauss points.

If the generalized internal element forces are not available, they can be computed from top and bottom stresses with equations (6) to (9), assuming a linear stress profile.

#### 4.1.5. Longitudinal reinforcements: analytical method

This method is activated by choosing “Analytical method” in the analysis settings (see User Manual [1]). The user can choose the increment in angle  $\theta$  (default value is  $5^\circ$ , meaning that the following angles are taken into account:  $0^\circ, 5^\circ, 10^\circ, \dots, 355^\circ$ ).

##### 4.1.5.1. Original method

This analytical method was developed by [4] and is sometimes referred to as the ‘Capra-Maury method’ or the facet rotation method.

The principle is to verify the equilibrium of a cross-section (so-called “facet”) perpendicular to the element, and whose normal has an angle  $\theta$  with the local  $x$ -axis (Figure 3), with  $\theta$  varying from 0 to  $2\pi$ . Equilibrium for all the facets implies equilibrium of the shell or plate element.

The external forces are a normal force  $N_\theta$  and a bending moment  $M_\theta$ . The resisting forces are the concrete compression stress, and the forces  $F_b(\theta)$  et  $F_t(\theta)$  in the bottom and top reinforcements; the methodology to determine the necessary resisting forces is given in chapter 4.2.

$$F_b(\theta) \leq R_b(\theta) \text{ et } F_t(\theta) \leq R_t(\theta). \quad (10)$$

The normal force and bending moment are obtained by rotating the tensor of generalized forces and moments:

$$N_\theta = N_{xx} \cos^2 \theta + N_{yy} \sin^2 \theta + 2N_{xy} \cos \theta \sin \theta, \quad (11)$$

$$M_\theta = M_{xx} \cos^2 \theta + M_{yy} \sin^2 \theta + 2M_{xy} \cos \theta \sin \theta. \quad (12)$$

Similarly, the resisting forces of reinforcements placed respectively along the  $x$ -axis and along the  $y$ -axis are obtained by:

$$A_x \sigma_x \cos^2 \theta, \quad (13)$$

$$A_y \sigma_y \sin^2 \theta, \quad (14)$$

where  $A_x$  and  $A_y$  are the reinforcement areas per unit length, and  $\sigma_x$  and  $\sigma_y$  are the stresses in the reinforcements.

Thus, the total resisting force of the bottom or top reinforcements is equal to

$$R(\theta) = A_x \sigma_x \cos^2 \theta + A_y \sigma_y \sin^2 \theta. \quad (15)$$

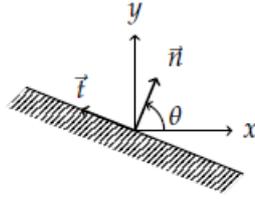


Figure 3 : Cross section (so-called “facet”) with angle  $\theta$  to the local  $x$ -axis.

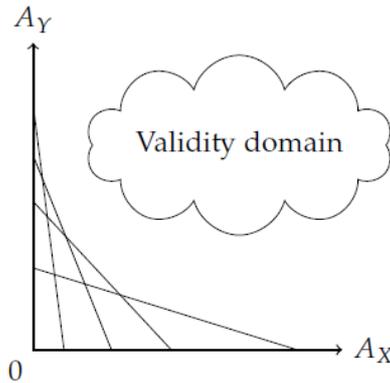


Figure 4 : Validity domain considering all angles  $\theta$ .

Equation (15) is valid for either the bottom or top reinforcements, with resisting force  $R_b(\theta)$  or  $R_t(\theta)$ . The equilibrium condition for the bottom or top reinforcements thus reads

$$A_x \sigma_x \cos^2(\theta) + A_y \sigma_y \sin^2(\theta) \geq F(\theta). \quad (16)$$

Each inequation for an angle  $\theta$  defines a straight line in the plane  $(A_x, A_y)$ . All the lines taken together define the validity domain in which  $A_x$  and  $A_y$  can be chosen (Figure 4). We note that the reinforcement stresses,  $\sigma_x$  and  $\sigma_y$ , can vary with the angle  $\theta$ .

The optimal solution is found by minimizing the total reinforcement area per unit length,  $A_x + A_y$ , with the constraints given by Equation (16), and  $A_x \geq 0$  and  $A_y \geq 0$ .

#### 4.1.5.2. Generalizations

##### Reinforcement angles

In the general case, the  $x$ -axis (resp.  $y$ -axis) reinforcement layer is not parallel to the local  $x$  (resp.  $y$ ) axis but has an angle  $\alpha$  (resp.  $\beta$ ) with it, see Figure 5.

Equation (15) is generalized as

$$R(\theta) = A_X \sigma_x \cos^2(\theta - \alpha) + A_Y \sigma_y \sin^2(\theta - \beta), \quad (17)$$

and Equation (16) becomes

$$A_X \sigma_x \cos^2(\theta - \alpha) + A_Y \sigma_y \sin^2(\theta - \beta) \geq F(\theta). \quad (18)$$

##### Reinforcement positions in cross section

The original algorithm assumes that both the  $x$ -axis and  $y$ -axis reinforcements are at the same position in the cross-section. The general case is handled in the following way.

We note  $c_x$  (resp.  $c_y$ ) the distance between concrete surface and reinforcement barycenter for the  $x$ -axis (resp.  $y$ -axis) reinforcements. For a given angle  $\theta$ , these reinforcements are equivalent to a single reinforcement layer at position  $c_\theta$ , provided that the global lever arm is unchanged:

$$R(\theta) \cdot c_\theta = (A_X \sigma_x \cos^2(\theta - \alpha)) \cdot c_x + (A_Y \sigma_y \sin^2(\theta - \beta)) \cdot c_y, \quad (19)$$

or equivalently

$$c_\theta = \frac{(A_X \sigma_x \cos^2(\theta - \alpha)) \cdot c_x + (A_Y \sigma_y \sin^2(\theta - \beta)) \cdot c_y}{A_X \sigma_x \cos^2(\theta - \alpha) + A_Y \sigma_y \sin^2(\theta - \beta)}. \quad (20)$$

The value depends on reinforcement areas per unit length  $A_X$  and  $A_Y$ , distances  $c_x$  and  $c_y$ , and stresses  $\sigma_x$  and  $\sigma_y$ .

For the practical implementation,  $c_\theta$  is computed by assuming  $A_X = A_Y$ , and by assuming that  $\sigma_x$  (resp.  $\sigma_y$ ) is the maximum allowable value  $\sigma_{a,x}$  (resp.  $\sigma_{a,y}$ ).

##### Different steel parameters in $x$ and $y$ directions

Steel allowable stress may be different for the  $x$ -axis and  $y$ -axis reinforcements. In this case the allowable stress of the equivalent reinforcement is given by

$$\sigma_{a,\theta} = \sigma_{a,x} \cos^2(\theta - \alpha) + \sigma_{a,y} \sin^2(\theta - \beta). \quad (21)$$

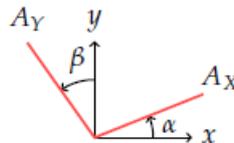


Figure 5 : Reinforcements positioned with an angle to the local axes.

#### **4.1.6. Longitudinal reinforcements: analytical method with envelope polygon**

This method will be added in a future version to provide performance enhancements.

## 4.2. EQUILIBRIUM OF A REINFORCED CONCRETE RECTANGULAR SECTION

### 4.2.1. Introduction

This chapter describes the methodology to justify the resistance of a rectangular reinforced concrete section to a normal force  $N$  and a bending moment  $M$ .

It is used for the automatic computation of steel reinforcements in shell elements, with the method described in chapter 4.1.5 or 4.1.6.

For clarity we show here the case of positive bending moment ( $M > 0$ ).

### 4.2.2. Definitions and conventions

We consider a rectangular reinforced concrete cross section with height  $h$  and width  $b$  (Figure 6). Top (resp. bottom) reinforcements have a section  $A_{s,t}$  (resp.  $A_{s,b}$ ) placed at distance  $c_{top}$  (resp.  $c_{bot}$ ) from the concrete surface. Lever arm is  $d_{top}$  (resp.  $d_{bot}$ ).

We introduce the distance between the two reinforcements,  $a = h - (c_{top} + c_{bot})$ , and the distances from the center of gravity of the cross section to the concrete surface,  $v_{bot}$  and  $v_{top}$ . In the case of a rectangular section,

$$v_{bot} = v_{top} = \frac{h}{2}. \quad (22)$$

A positive normal force ( $N > 0$ ) indicates compression. A positive bending moment ( $M > 0$ ) produces tension in the bottom reinforcements (Figure 6).

The load is equivalent to a normal force  $N$  applied with an eccentricity  $e_0$  given by

$$e_0 = \frac{M}{N}. \quad (23)$$

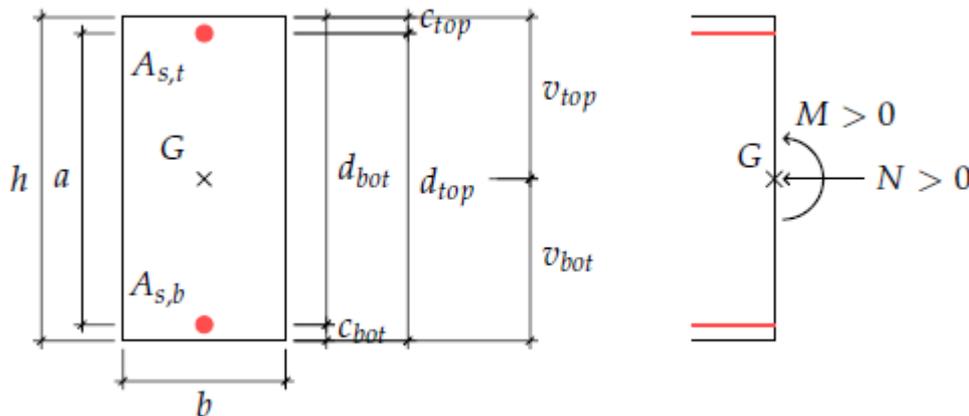


Figure 6 : Rectangular cross-section.

### 4.2.3. Equilibrium equations

External forces are resisted by:

- $N_c \geq 0$  : compression force in concrete;
- $N_{s,b}$  : tension ( $\leq 0$ ) or compression ( $> 0$ ) in bottom reinforcements;
- $N_{s,t}$  : tension ( $\leq 0$ ) or compression ( $> 0$ ) in top reinforcements.

The bending moment at bottom reinforcement position is

$$M_{/A_{s,b}} = N \cdot (v_{bot} - c_{bot}) + M = N \cdot e_b, \quad (24)$$

where  $e_b = v_{bot} - c_{bot} + e_0$  is the eccentricity of the normal force with respect to bottom reinforcements.

The equilibrium conditions at bottom reinforcement position thus read:

$$N = N_c + N_{s,b} + N_{s,t} \quad (25)$$

$$M_{/A_{s,b}} = N \cdot e_b = N_c \cdot z + N_{s,t} \cdot a. \quad (26)$$

The software computes the equilibrium of the section and determines the state of the section (strain and stress profile in concrete and reinforcements), as well as the reinforcement areas  $A_{s,b}$  et  $A_{s,t}$  that are sufficient to satisfy Equations (25) and (26).

Design criteria are enforced using behavior laws specific to the limit state and to the design code used (refer to the following chapters).

When several choices of  $A_{s,b}$  and  $A_{s,t}$  are possible, an optimization is performed to minimize the total area  $A_{s,b} + A_{s,t}$ .

Call  $\alpha = x_u/d_{bot}$  the height of concrete in compression divided by the bottom reinforcement lever arm. Depending on the value of  $\alpha$ , the cross-section can be in one of the following three states:

1. Section is entirely in tension: in this case the only resisting forces are tension forces in the reinforcements.
2. Section is partly in compression and partly in tension: in this case the equilibrium is obtained by using both concrete and reinforcements. For high values of  $\alpha$ , the stress in bottom reinforcement may become less than yield stress, in which case it may be more economical or even necessary to use top reinforcements in compression.
3. Section is entirely in compression: in this case the equilibrium is generally obtained by concrete alone without reinforcements. However, if the concrete stress or strain exceed the design criteria, the software computes the bottom and top reinforcement areas that are necessary to remain within the design criteria.

#### 4.2.4. Available behavior laws

##### 4.2.4.1. Ultimate Limit State (ULS)

###### Main assumptions

The ultimate limit state equilibrium is based on the following assumptions:

- cross section remains plane,
- perfect steel-concrete bonding,
- concrete resistance in tension is neglected,
- nonlinear behavior laws for both concrete and reinforcements.

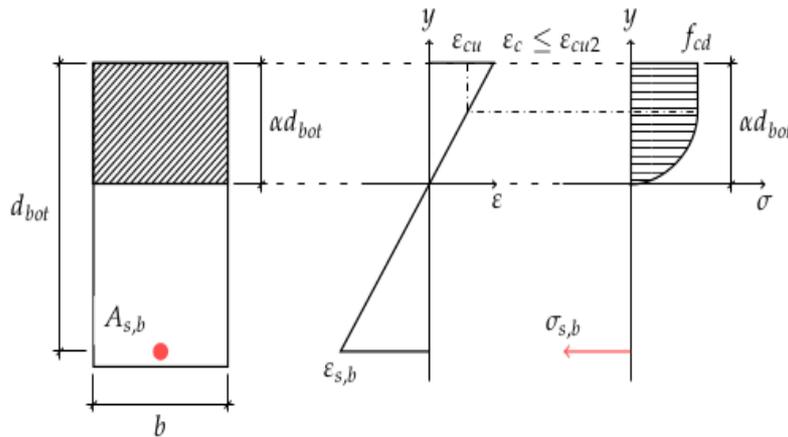


Figure 7 : Example Ultimate Limit State equilibrium.

###### Reinforcement behavior law

An elastic-perfect plastic law is used for reinforcements, without deformation limit.

The parameters are the reinforcement Young's modulus  $E_s$  (default value 200 000 MPa) and allowable stress  $\sigma_{a,b}$  for bottom reinforcements and  $\sigma_{a,t}$  for top reinforcements.

Yield strain is computed by

$$\varepsilon_y = \frac{\sigma_a}{E_s}. \quad (27)$$

###### Concrete behavior law

The parabola-rectangle law defined by Eurocode 2 [2] is used for concrete:

$$\sigma(\varepsilon) = \begin{cases} f_{cd} \left[ 1 - \left( 1 - \frac{\varepsilon}{\varepsilon_{c2}} \right)^n \right], & 0 \leq \varepsilon \leq \varepsilon_{c2} \\ f_{cd}, & \varepsilon_{c2} < \varepsilon \leq \varepsilon_{cu2} \\ 0, & \text{otherwise} \end{cases} \quad (28)$$

The parameters are:

- $f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$ : concrete design resistance in compression,

- $\varepsilon_{c2}$ : strain corresponding to maximal stress (default value 0.2%),
- $\varepsilon_{cu2}$ : ultimate design strain (default value 0.35%),
- $n$ : exponent (default value 2).

#### 4.2.4.2. Serviceability Limit State (SLS)

##### Main assumptions

The service limit state equilibrium is based on the following assumptions:

- cross section remains plane,
- perfect steel-concrete bonding,
- concrete resistance in tension is neglected,
- linear behavior laws for both concrete and reinforcements.

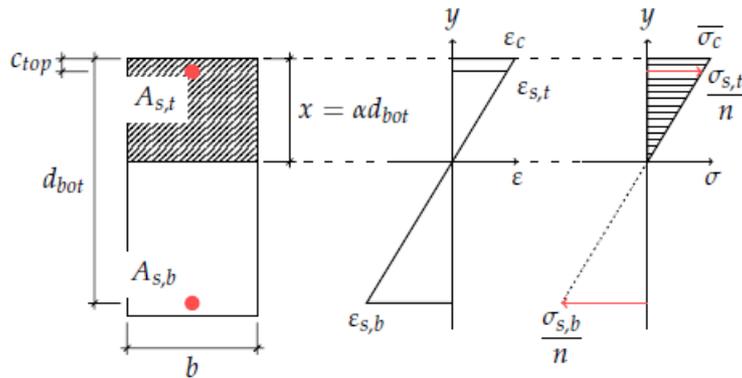


Figure 8 : Example Serviceability Limit State equilibrium.

##### Reinforcement behavior law

A linear elastic behavior law is used for reinforcements, with an upper and lower bound on stress.

The parameters are the reinforcement Young's modulus  $E_s$  (default value 200 000 MPa) and allowable stress  $\sigma_{a,b}$  for bottom reinforcements and  $\sigma_{a,t}$  for top reinforcements.

##### Concrete behavior law

A linear elastic law is used for concrete in compression. The slope is given by the equivalent Young's modulus:

$$E_{c,eff} = \frac{E_s}{n} \quad (29)$$

where  $n$  is an equivalence coefficient.

The compression stress is limited to the allowable stress  $\sigma_{c,max}$ .

## 5. TRANSVERSE REINFORCEMENTS

### 5.1. MAXIMUM TRANSVERSE SHEAR FORCE

The two transverse shear forces per unit length are used to determine the transverse shear force per unit length for each angle  $\theta$  (see also chapter 4.1.5),

$$Q_{\theta} = Q_x \cos \theta + Q_y \sin \theta, \quad (30)$$

which is used to verify the resistance according to the following methodology.

### 5.2. EUROCODE 2

Resistance to transverse shear force is verified according to Eurocode 2 [2] chapter 6.2.

The computation results are given in the following units:

- transverse reinforcement area per unit area:  $cm^2/m^2$ ,
- concrete shear stress:  $MPa$ .

#### 5.2.1. Notations

The shear force is rewritten more conveniently in the form of an equivalent shear stress,

$$\tau_u = \frac{|V_{Ed}|}{b_w \cdot d} \quad (31)$$

where  $b_w = 1 m$  (unit width) and  $d$  is  $d_{bot}$  or  $d_{top}$  depending on the sign of the bending moment.

#### 5.2.2. Resistance of concrete alone

The design shear resistance of concrete,  $V_{Rd,c}$ , is computed according to article 6.2.2 Equation (6.2) rewritten below in the form of a shear stress:

$$\tau_{Rd,c} = \max(v; v_{min}) + k_1 \sigma_{cp} \quad (32)$$

$$v = C_{Rd,c} \cdot k \cdot (100 \rho_l f_{ck})^{\frac{1}{3}} \quad (33)$$

where:

- the mean compression stress is  $\sigma_{cp} = N/A_c$ , with upper bound  $0,2 f_{cd}$ ,
- $A_c$  is the area of the concrete section,
- the height  $h$  is equal to the thickness of the shell or plate element,
- $\rho_l$  is computed from the reinforcement areas computed in chapter 4 for the current *combination*, as the minimum value between  $A_x$  and  $A_y$ . These

reinforcements are supposed to be anchored at a distance  $l_{bd} + d$ . When  $M \geq 0$ , the bottom reinforcements are used; when  $M < 0$ , the top reinforcements are used. In both cases  $\rho_l$  has an upper bound of 2%,

- the recommended values for  $k$ ,  $C_{Rd,c}$  et  $k_1$  are used:

$$k_1 = 0,15 \quad (34)$$

$$k = \min\left(1 + \sqrt{\frac{0,2}{d}}; 2\right) \quad (35)$$

$$C_{Rd,c} = \frac{0,18}{\gamma_c} \quad (36)$$

- $v_{min}$  is defined according to national annex (NF EN 1992-1-1/NA §6.2.2 (1) NOTE), with a formula chosen by the user (See *User Manual [1]*, property “*Transverse Redistribution*”):

- *Slabs with transverse redistribution* ( $flag\_v = 0$ ):

$$v_{min} = 0,23 \cdot f_{ck}^{1/2}$$

- *Beams and slabs without transverse redistribution* ( $flag\_v = 1$ ):

$$v_{min} = 0,035 \cdot k^{3/2} \cdot f_{ck}^{1/2}$$

- *Walls* ( $flag\_v = 2$ ):

$$v_{min} = 0,23 \cdot f_{ck}^{1/2}$$

When  $V_{Ed} \leq V_{Rd,c}$  and  $V_{Ed} \leq V_{Rd,max}$  computed according to Equation (6.5), transverse reinforcements are not necessary. If any of these two conditions is not satisfied, necessary transverse reinforcements are computed according to the following chapter.

Equation (6.5) is reproduced below in the form of a maximum equivalent shear stress

$$\tau_{Rd,max} = 0,5 \cdot v \cdot f_{cd} \quad (37)$$

with

$$v = 0.6 \left(1 - \frac{f_{ck}}{250}\right) \quad (38)$$

### 5.2.3. Resistance with transverse reinforcements

#### 5.2.3.1. Verification of compression stress in concrete struts

Compression stress in concrete struts is verified by comparing the acting force  $V_{Ed}$  to the resisting force  $V_{Rd,max}$ , computed according to article 6.2.3 Equation (6.9), rewritten below in the form of a maximum equivalent shear stress:

$$\tau_{Rd,max} = \frac{0,9 \cdot \alpha_{cw} \cdot v_1 \cdot f_{cd}}{\cot(\theta) + \tan(\theta)} \quad (39)$$

where:

- the lever arm  $z$  is taken equal to  $0,9 d$ ,
- the angle of transverse rebars is  $\alpha = 90^\circ$ ,
- the angle of concrete struts is  $\theta = 45^\circ$ ,
- the coefficient  $v_1$  that reduces the resistance of cracked concrete to shear forces is computed as follows (See User Manual [1], property “Method Calculation For  $nu_1$ ”):
  - Equal to  $nu$  (`flag_nu = 0`): apply article 6.2.3(3) NOTE 1, giving  $v_1 = v$ ;
  - Alternative method (`flag_nu = 1`) apply article 6.2.3(3) NOTE 2 Equations (6.10aN) and (6.10bN), giving:
    - $v_1 = 0,6$  for  $f_{ck} \leq 60 \text{ MPa}$
    - $v_1 = \max\left(0,9 - \frac{f_{ck}}{200}; 0,5\right)$  for  $f_{ck} > 60 \text{ MPa}$
 and use  $0.8 f_{ywd}$  instead of  $f_{ywd}$  in §5.2.3.2,
- the coefficient  $\alpha_{cw}$  is defined according to national annex recommendations (NF EN 1992-1-1/NA §6.2.3 (3) NOTES 1 à 3):
  - $\alpha_{cw} = 1$  if  $0 \leq \sigma_{cp}$
  - $\alpha_{cw} = 1 + \frac{\sigma_{cp}}{f_{ctm}}$  if  $-f_{ctm} \leq \sigma_{cp} < 0$
  - $\alpha_{cw} = 0$  if  $\sigma_{cp} < -f_{ctm}$

For the specific case of a section entirely in tension, the coefficient is 0 and thus  $V_{Rd,max} = V_{Rd,c} = 0$  (this case is not handled by Eurocode).

### 5.2.3.2. Necessary transverse reinforcements

The necessary transverse reinforcement area per unit area is computed according to article 6.2.3 Equation (6.8), which we reproduce below using equivalent shear stress:

$$\frac{A_{sw}}{s} = \frac{b_w \cdot \tau_u}{0,9 \cdot f_{ywd} \cdot \cot(\theta)} \quad (40)$$

If the compression stress of concrete struts is not verified (see §5.2.3.1), the transverse reinforcement area is not computed and is replaced by value  $999 \text{ cm}^2/\text{m}^2$  (error code).

### 5.2.4. Shear combined with tension

In the case of shear combined with tension, the coefficient  $\alpha_{cw}$  is equal to 0 (see §5.2.3.1). The resistance of the section cannot be assessed by using the Eurocode equations.

In this case, the transverse reinforcement area is replaced by an arbitrary value  $999 \text{ cm}^2/\text{m}^2$  (error code).

### 5.2.5. Results

The following results are available as fields in Ansys Mechanical:

- transverse reinforcement area per unit area ( $\frac{A_{sw}}{s}$ ), see §5.2.3.2;
- concrete equivalent shear stress ( $\tau_u$ ), see §5.2.1;
- concrete resistant shear stress ( $\tau_{Rd,c}$ ), see §5.2.2;
- concrete maximal shear stress ( $\tau_{Rd,max}$ ), see §5.2.2 and §5.2.3.1.

## **6. BIBLIOGRAPHY**

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